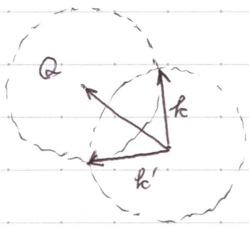
引力相互作用

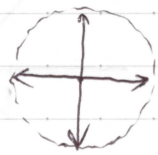
$$-\frac{1}{2} V c^{\dagger}_{k+q\sigma} c^{\dagger}_{k'-q\sigma'} c_{k'\sigma'} c_{k\sigma}$$

$$(k, k') \longrightarrow (k+q, k'-q)$$

$$k + k' = Q$$



$k + k' = 0$



$k' = -k$

σ, σ' $\quad \uparrow \downarrow$ $\quad$ 引力をかせげる

$$-\frac{1}{2} V \sum_{k,k'} \left\{ c^{\dagger}_{k'\uparrow} c^{\dagger}_{-k'\downarrow} c_{-k\downarrow} c_{k\uparrow} + c^{\dagger}_{k'\downarrow} c^{\dagger}_{-k'\uparrow} c_{-k\uparrow} c_{k\downarrow} \right\}$$

$$= -V \sum_{\substack{|\xi_k| < \hbar\omega_D \\ |\xi_{k'}| < \hbar\omega_D}} c^{\dagger}_{k'\uparrow} c^{\dagger}_{-k'\downarrow} c_{-k\downarrow} c_{k\uparrow}$$

$V = V \quad |\xi_k|, |\xi_{k'}| < \hbar\omega_D$

$V = 0 \quad$ otherwise

$$H_{BCS} = \sum_{k\sigma} \xi_k c^{\dagger}_{k\sigma} c_{k\sigma} - V \sum_{\substack{|\xi_k| < \hbar\omega_D \\ |\xi_{k'}| < \hbar\omega_D}} c^{\dagger}_{k'\uparrow} c^{\dagger}_{-k'\downarrow} c_{-k\downarrow} c_{k\uparrow}$$

$$\underline{\xi_k = \varepsilon_k - \mu}$$

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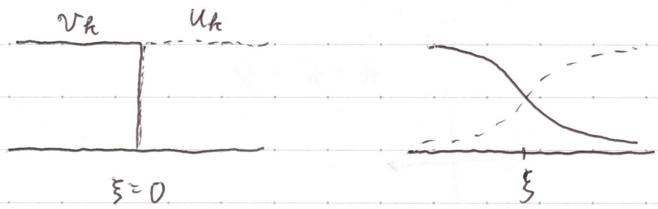
3.8 BCS 基底状態

BCS 波動関数

$$|\Psi_{BCS}\rangle = \prod_k \left(\underbrace{u_k}_{\text{いない}} + \underbrace{v_k}_{\text{いる}} c^\dagger_{k\uparrow} c^\dagger_{-k\downarrow}\right)|0\rangle$$



$$\frac{v_k}{u_k} = \alpha \underline{\underline{e^{i\varphi}}}$$ 位相

秩序変数（超伝導ギャップ）

$$\langle c_{-k\downarrow} c_{k\uparrow} \rangle$$

$$\Delta = V \sum_k \langle c_{-k\downarrow} c_{k\uparrow} \rangle$$

$$= V \sum_k u_k^* v_k$$

$$\Delta^* = V \sum_k \langle c^\dagger_{k\uparrow} c^\dagger_{-k\downarrow} \rangle$$

平均場近似

$$c_k^\dagger c_{-k}^\dagger = \langle c_k^\dagger c_{-k}^\dagger \rangle + \overset{\text{ゆらぎ}}{\{ c_k^\dagger c_{-k}^\dagger - \langle c_k^\dagger c_{-k}^\dagger \rangle \}}$$

$$c_{-k} c_k = \langle c_{-k} c_k \rangle + \{ c_{-k} c_k - \langle c_{-k} c_k \rangle \}$$

ゆらぎ×ゆらぎ～0

$$- V \sum_{k,k'} c_{k'\uparrow}^\dagger c_{-k'\downarrow}^\dagger c_{-k\downarrow} c_{k\uparrow}$$

$$= V \sum_{k,k'} \langle c_{k'\uparrow}^\dagger c_{-k'\downarrow}^\dagger \rangle \langle c_{-k\downarrow} c_{k\uparrow} \rangle$$

$$- V \sum_{k,k'} \langle c_{k'\uparrow}^\dagger c_{-k'\downarrow}^\dagger \rangle c_{-k\downarrow} c_{k\uparrow}$$

$$- V \sum_{k,k'} \langle c_{-k\downarrow} c_{k\uparrow} \rangle c_{k'\uparrow}^\dagger c_{-k'\downarrow}^\dagger$$

$$= + \frac{|\Delta|^2}{V} + \sum_k \left(-\Delta^* c_{-k\downarrow} c_{k\uparrow} - \Delta c_{k\uparrow}^\dagger c_{-k\downarrow}^\dagger \right)$$

準粒子 (Bogoliubov 変換)

$$\alpha_{k\uparrow} = u_k c_{k\uparrow} - v_k c_{-k\downarrow}^\dagger$$

$$\alpha_{-k\downarrow}^\dagger = u_k^* c_{-k\downarrow}^\dagger + v_k^* c_{k\uparrow}$$

$|u_k|^2 + |v_k|^2 = 1$ とすると、

$$\{ \alpha_{k\sigma}, \alpha_{k'\sigma'}^\dagger \} = \delta_{kk'} \delta_{\sigma\sigma'}$$

$$\{ \alpha_{k\sigma}, \alpha_{k'\sigma'} \} = 0$$

逆変換

$$c_{k\uparrow} = u_k^* \alpha_{k\uparrow} + v_k \alpha_{-k\downarrow}^\dagger$$

$$c_{-k\downarrow}^\dagger = u_k \alpha_{-k\downarrow}^\dagger - v_k^* \alpha_{k\uparrow}$$

平均場ハミルトニアンに代入

$$\overbrace{\{\xi_k(|u_k|^2-|v_k|^2)+\Delta^* u_k^* v_k+\Delta u_k v_k^*\}}^{E_k}\alpha_{k\uparrow}^\dagger\alpha_{k\uparrow}$$

$$+\{\xi_k(|u_k|^2-|v_k|^2)+\Delta^* u_k^* v_k+\Delta u_k v_k^*\}\alpha_{-k\downarrow}^\dagger\alpha_{-k\downarrow}$$

$$+(2\xi_k u_k v_k+\Delta^* v_k^2-\Delta u_k^2)\alpha_{k\uparrow}^\dagger\alpha_{-k\downarrow}^\dagger$$

$$+(2\xi_k u_k^* v_k^*-\Delta^* u_k^{*2}+\Delta v_k^{*2})\alpha_{-k\downarrow}\alpha_{k\uparrow}$$

共役

$$+\underbrace{\left(\frac{|\Delta|^2}{V}+2\xi_k|v_k|^2-\Delta^* u_k^* v_k-\Delta u_k v_k^*\right)}_{E_0}$$

$\alpha_{k\uparrow}^\dagger\alpha_{-k\downarrow}^\dagger$ の係数 ($=\alpha_{-k\downarrow}\alpha_{k\uparrow}$ の係数) を 0 にしたい;

$$\Delta^* v_k^2-\Delta u_k^2+2\xi_k u_k v_k=0$$

$$\frac{v_k}{u_k}=\alpha e^{i\varphi},\quad \Delta=\Delta_0 e^{i\phi}$$

$-\phi+2\varphi \qquad \phi \qquad \varphi$

$$\boxed{\phi=\varphi}$$

以下、$\varphi=0$ とする。

$$\begin{cases} u_k=\cos\theta_k \\ v_k=\sin\theta_k \end{cases}$$

$$-\Delta\cos 2\theta_k+\xi_k\sin 2\theta_k=0$$

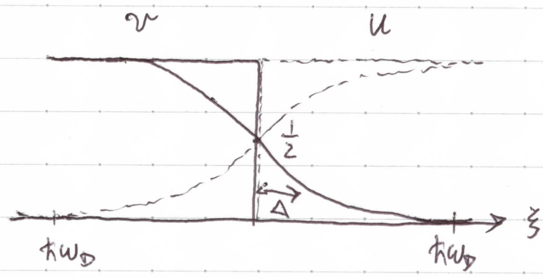
$$\tan 2\theta_k=\frac{\Delta}{\xi_k}$$

$$\begin{cases} \cos 2\theta_k=u^2-v^2=\dfrac{\xi_k}{\sqrt{\Delta^2+\xi_k^2}} \\ \sin 2\theta_k=2uv=\dfrac{\Delta}{\sqrt{\Delta^2+\xi_k^2}} \\ u^2+v^2=1 \end{cases}$$

$\xi_k>0$ で $+$

$$u^2=\frac{1}{2}\left(1+\frac{\xi_k}{\sqrt{\Delta^2+\xi_k^2}}\right)$$

$$v^2=\frac{1}{2}\left(1-\frac{\xi_k}{\sqrt{\Delta^2+\xi_k^2}}\right)$$



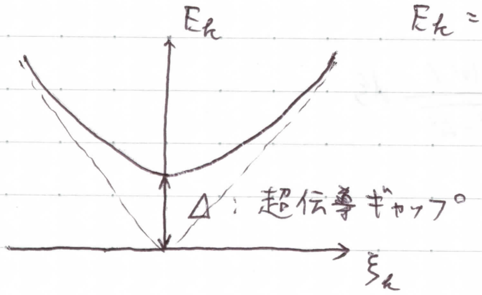
励起エネルギー

$$H = E_0 + \sum_k E_k \left(\alpha^{+}_{k\uparrow} \alpha_{k\uparrow} + \alpha^{+}_{-k\downarrow} \alpha_{-k\downarrow} \right)$$

$$E_k = \xi (u_k^2 - v_k^2) + 2\Delta u_k v_k$$
$$= \sqrt{\Delta^2 + \xi_k^2}$$

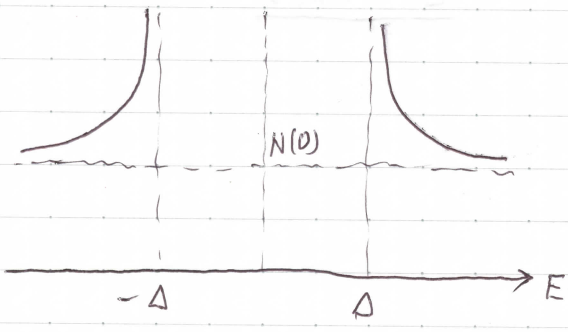
基底状態

$$\left.\begin{aligned} \alpha_{k\uparrow} |\Phi_G\rangle = 0 \\ \alpha_{-k\downarrow} |\Phi_G\rangle = 0 \end{aligned}\right\} \text{BCS みたしている}$$



$$E_k = \sqrt{\xi_k^2 + \Delta^2}$$

$$D = N(0) \frac{d\xi_k}{dE_k} = \frac{N(0)\, E_k}{\sqrt{E_k^2 - \Delta^2}}$$



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Δ, T_c の決定

$$\Delta = V\sum_k \langle c_{-k\downarrow} c_{k\uparrow} \rangle$$

$$= V\sum_k \{ u_k v_k - u_k v_k \langle \alpha^\dagger_{-k\downarrow}\alpha_{-k\downarrow}\rangle - u_k v_k \langle \alpha^\dagger_{k\uparrow}\alpha_{k\uparrow}\rangle \}$$

$\langle \alpha^\dagger_{k\uparrow}\alpha_{k\uparrow}\rangle = f(E_k)$; フェルミ分布関数

$$= V\sum_k u_k v_k (1 - 2f(E_k))$$

$$= \frac{V}{2}\sum_k \frac{\Delta}{E_k}\tanh\frac{\beta E_k}{2}$$; ギャップ方程式

$T \to 0$, $\beta \to \infty$

$$1 = \frac{V}{2}\sum_k \frac{1}{E_k}$$

$$= \int_{-\hbar\omega_D}^{\hbar\omega_D} \frac{N(0)V}{2\sqrt{\xi^2+\Delta^2}} d\xi = 2\int_0^{\hbar\omega_D} \frac{N(0)V}{2\sqrt{\xi^2+\Delta^2}} d\xi$$

$$= N(0)V \sinh^{-1}\frac{\hbar\omega_D}{\Delta}$$

$$\sinh\left(\frac{1}{N(0)V}\right) = \frac{\hbar\omega_D}{\Delta}$$

$$\Delta = \hbar\omega_D \frac{1}{\sinh\left(\frac{1}{N(0)V}\right)} \sim 2\hbar\omega_D \exp\left(-\frac{1}{NV}\right)$$

$NV \ll 1$

$\hbar\omega_D \sim 10\,\mathrm{meV}$

$NV \sim 0.33 \to e^{-3} \sim \frac{1}{20}$ $\qquad$ $\underline{\Delta \sim 1\,\mathrm{meV}}$

T_c, β_c, $E_k \to \xi$

$$1 = V\sum_k \frac{1}{2\xi_k}\tanh\frac{\beta_c \xi_k}{2}$$

$$= N(0)V\int_{-\hbar\omega_D}^{\hbar\omega_D} \frac{\tanh\frac{\beta_c\xi}{2}}{2\xi} d\xi$$

$$= NV\int_0^{\frac{1}{2}\beta_c\hbar\omega_D} \frac{\tanh x}{x} dx$$

$$\frac{1}{NV} = \ln\left(\frac{2e^\gamma}{\pi}\beta_c\hbar\omega_D\right)$$, γ : オイラー定数

$$\exp\left(\frac{1}{N(0)V}\right) = \frac{2e^{\gamma}}{\pi}\frac{1}{k_B T_c}\hbar\omega_D$$

$$\underline{k_B T_c = 1.13\,\hbar\omega_D \exp\left(-\frac{1}{N(0)V}\right)}$$

$$\underline{\frac{\Delta}{k_B T_c} = 1.57} \qquad \frac{2\Delta}{k_B T_c} = 3.14$$

基底状態のエネルギー

$$E_0 = \frac{|\Delta|^2}{V} + \sum_k \left(2\xi_k|v_k|^2 - \Delta^* u_k^* v_k - \Delta u_k v_k^*\right)$$

$$\Delta^* = V\sum_k \langle c_{k\uparrow}^\dagger c_{-k\downarrow}^\dagger\rangle = V\sum_k u_k v_k^*$$

$$= \sum_k \left(\Delta u_k v_k^* + 2\xi_k|v_k|^2 - \Delta^* u_k^* v_k - \Delta u_k v_k^*\right)$$

$$= \sum_k \left(2\xi_k v_k^2 - \Delta u_k v_k\right)$$

$$= \sum_k \left(\xi_k - \frac{\xi_k^2}{E_k} - \frac{\Delta^2}{2E_k}\right) \equiv E_{super}$$

一方,

$$E_0(\Delta=0) = \sum_k \left(\xi_k - \frac{\xi_k^2}{|\xi_k|}\right) = \sum_{\xi_k<0} 2\xi_k \equiv E_{normal}$$

$$E_{super} - E_{normal}$$

$$= \sum_{\xi_k>0}\left(\xi_k - \frac{\xi_k^2}{E_k} - \frac{\Delta^2}{2E_k}\right) + \sum_{\xi_k<0}\left(-\xi_k - \frac{\xi_k^2}{E_k} - \frac{\Delta^2}{2E_k}\right)$$

$$= \sum_{\xi_k>0}\left(2\xi_k - \frac{2\xi_k^2}{E_k} - \frac{\Delta^2}{E_k}\right)$$

$$= \sum_{\xi_k>0}\left(2\xi_k - \frac{\xi_k^2}{E_k} - E_k\right)$$

$$= \int_0^\infty \left(2\xi - (\xi E)'\right) N(0)\, d\xi$$

$$= \left[\xi^2 - \underbrace{\xi E}_{\xi^2\left(1 + \frac{1}{2}\frac{\Delta^2}{\xi^2} + O(\xi^{-4})\right)}\right]_0^\infty N(0)$$

$$= -\frac{1}{2}N(0)\Delta^2$$

$$E_G = -\frac{1}{2}N(0)\Delta^2 = -\frac{(1.57)^2}{2}N(0)(k_B T_c)^2$$

$$\sim -1.25\,N(0)(k_B T_c)^2$$