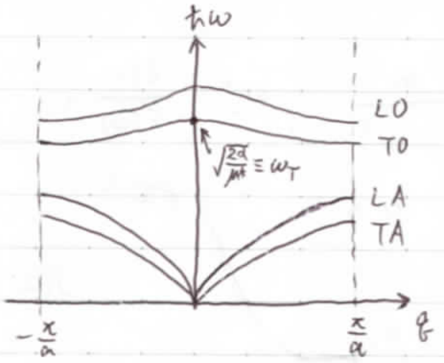
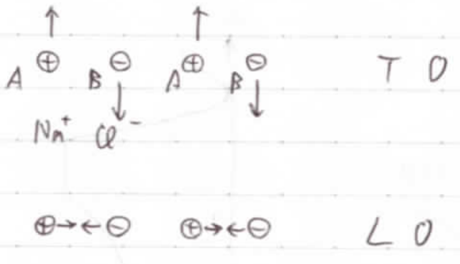
1.5 光学モードと光の相互作用



$$\frac{1}{\mu^*} \equiv \frac{1}{M_A} + \frac{1}{M_B}$$



TO フォノン　光と couple

$$\begin{cases} -\omega^2 M_A u_A = -2\alpha(u_A - u_B) + eE \\ -\omega^2 M_B u_B = -2\alpha(u_B - u_A) - eE \end{cases}$$

$$-\omega^2(u_A - u_B) = -\frac{2\alpha}{\mu^*}(u_A - u_B) + \frac{eE}{\mu^*}$$

$$\underset{\displaystyle P}{\underset{\parallel}{e(u_A - u_B)}} = \frac{e^2/\mu^*}{\omega_T^2 - \omega^2} E$$

$$D = \varepsilon_0 E + P$$

$$\downarrow$$

$$\underbrace{\varepsilon(\infty)}_{\text{他の寄与を考慮}} E + P = \varepsilon(\omega) E$$

$$\varepsilon(\omega) = \varepsilon(\infty) + \frac{e^2/\mu^*}{\omega_T^2 - \omega^2} = \varepsilon(\infty) + (\varepsilon(0) - \varepsilon(\omega)) \frac{\omega_T^2}{\omega_T^2 - \omega^2}$$

$$\left[\begin{array}{l} \varepsilon(0) = \varepsilon(\infty) + \dfrac{e^2/\mu^*}{\omega_T^2} \\ \rightarrow \ e^2/\mu^* = (\varepsilon(0) - \varepsilon(\infty))\,\omega_T^2 \end{array} \right]$$

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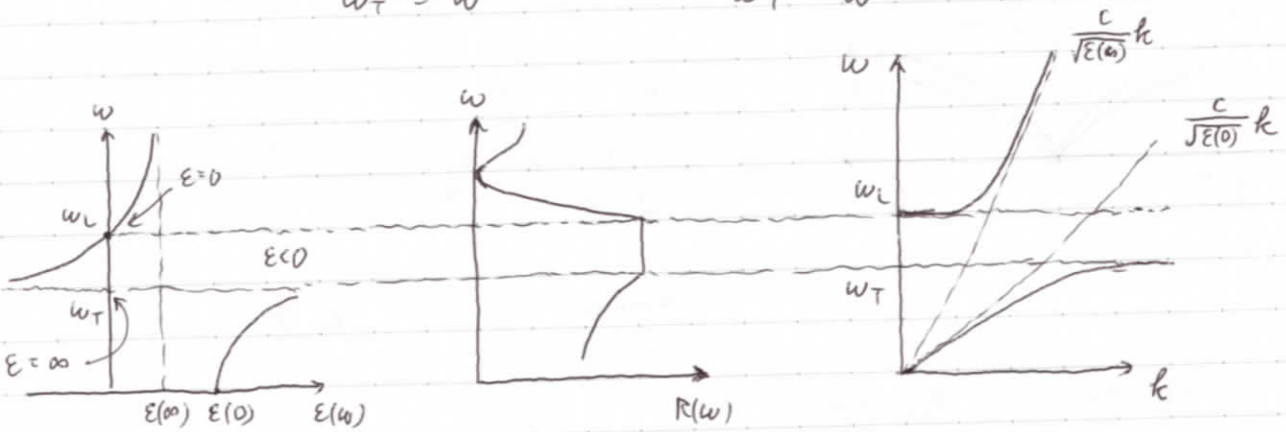
$$= \frac{\varepsilon(0)\,\omega_T^2 - \varepsilon(\infty)\,\omega^2}{\omega_T^2 - \omega^2}$$

$$= \varepsilon(\infty)\,\frac{\left(\frac{\varepsilon(0)}{\varepsilon(\infty)}\,\omega_T^2 - \omega^2\right)}{\omega_T^2 - \omega^2} = \varepsilon(\infty)\,\frac{\omega_L^2 - \omega^2}{\omega_T^2 - \omega^2}, \quad \omega_L^2 = \frac{\varepsilon(0)}{\varepsilon(\infty)}\,\omega_T^2$$



cubic → $\mathbb{E} \,/\!/\, \mathbb{D} \,/\!/\, \mathbb{P}$

$$\begin{cases} \nabla\cdot\mathbb{D} = 0 \\ \nabla\times\mathbb{E} = \nabla\times(\nabla\phi) = 0 \end{cases}$$

→ 「$\mathbb{D} = 0$ or $k\cdot\mathbb{D} = 0$」

かつ「$\mathbb{E} = 0$ or $k\times\mathbb{E} = 0$」

T $\quad k \perp \mathbb{P} \rightarrow \mathbb{E} = 0 \rightarrow \varepsilon = \infty$

L $\quad k \,/\!/\, \mathbb{P} \rightarrow \mathbb{D} = 0 \rightarrow \varepsilon = 0$

$T = 300\,\mathrm{K}$ のとき

$$\omega = \frac{k_B T}{\hbar} = \frac{4\times 10^{-21}\,\mathrm{J}}{\hbar} \sim 4\times 10^{13}\,\mathrm{s}^{-1}$$

$$k \sim \frac{\omega}{c} \sim 10^{5}\,\mathrm{m}^{-1}$$

$$\overset{\wedge}{\wedge}$$

$$\frac{\pi}{a} \sim \frac{1}{10^{-10}} \sim 10^{10}\,\mathrm{m}^{-1}$$

$\omega_L^2 = \frac{\varepsilon(0)}{\varepsilon(\infty)}\,\omega_T^2$; Lyddane – Sachs – Teller relation

1.6 格子比熱

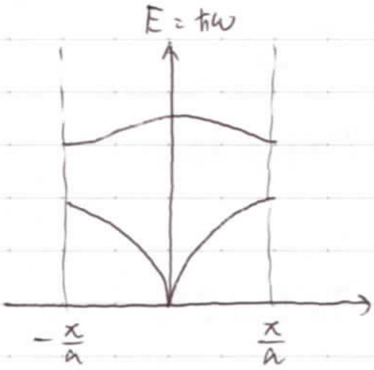
調和振動子の集まり

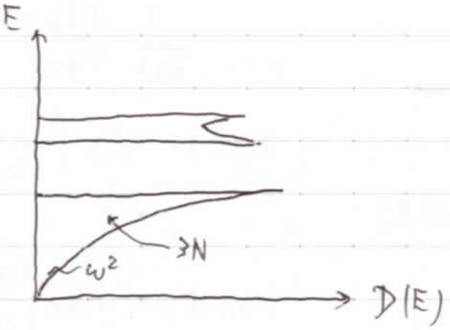
$$E = \sum_{k\alpha} \hbar\omega_{k\alpha}\left(b^{+}_{k\alpha} b_{k\alpha} + \frac{1}{2}\right)$$

$$= \sum_{k\alpha} \hbar\omega_{k\alpha}\left(n_{k\alpha} + \frac{1}{2}\right)$$

BE 統計に従う ; $\frac{1}{e^{E/k_BT} - 1}$

量子化「フォノン」





$$\varepsilon(T) = \int_0^\infty \frac{E\,D(E)}{e^{\frac{E}{k_BT}} - 1}\,dE$$

$$C_v(T) = \frac{d\varepsilon}{dT} = \int_0^\infty \frac{\frac{E^2}{k_BT^2}\,e^{\frac{E}{k_BT}}\,D(E)}{\left(e^{\frac{E}{k_BT}} - 1\right)^2}\,dE$$

T → ∞

$$C = k_B\int_0^\infty D(E)\,dE = 3Nk_B$$

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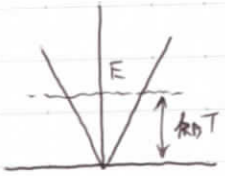
No.

Date

$T \to 0$

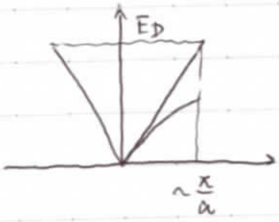


低エネルギーのモードのみ

$$\omega = vk$$

$$k = \frac{\omega}{v} = \frac{E}{\hbar v}$$

$$n(E) = \frac{1}{8\pi^3}\frac{4\pi}{3}\left(\frac{E}{\hbar v}\right)^3 = \frac{1}{6\pi^2}\left(\frac{E}{\hbar v}\right)^3$$



$$n(E_D) = N_A$$

$$\frac{1}{6\pi^2}\left(\frac{E_D}{\hbar v}\right)^3 = N_A$$

$$n(E) = N_A\left(\frac{E}{E_D}\right)^3$$ ← 1ブランチあたり

$$\Big\downarrow \times 3$$

$$n_{TOT}(E) = 3N_A\left(\frac{E}{E_D}\right)^3$$ TとLは区別

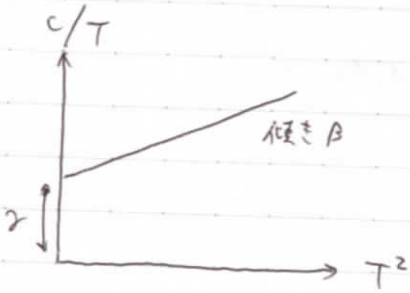
$$D(E) = 9N_A\frac{E^2}{E_D{}^3} \propto \omega^2$$

$$\therefore C_v(T) = \int_0^\infty \frac{\frac{E^2}{k_BT^2}e^{\frac{E}{k_BT}}\,9N_A\frac{E^2}{E_D{}^3}}{\left(e^{\frac{E}{k_BT}}-1\right)^2}dE$$

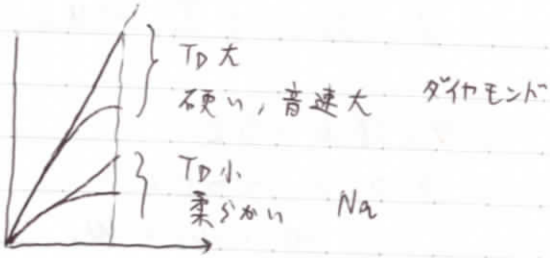
$$= 9R\left(\frac{k_BT}{E_D}\right)^3\int_0^\infty \frac{\left(\frac{E}{k_BT}\right)^4 e^{\frac{E}{k_BT}}}{\left(e^{\frac{E}{k_BT}}-1\right)^2}d\left(\frac{E}{k_BT}\right)$$, $R = N_Ak_B$

$$= 9R\left(\frac{T}{T_D}\right)^3\underbrace{\int_0^\infty \frac{x^4e^x}{(e^x-1)^2}dx}_{=\frac{4}{15}\pi^4}$$, $x \equiv \frac{E}{k_BT}$, $E_D = k_B\underset{\text{Debye温度}}{\underset{\sim}{T_D}}$

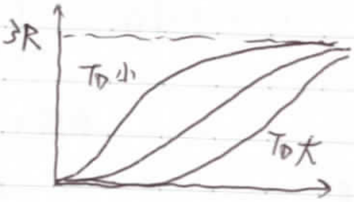
$$= \frac{12\pi^4}{5}R\left(\frac{T}{T_D}\right)^3 = 234R\left(\frac{T}{T_D}\right)^3$$

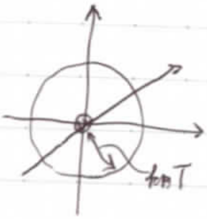


$$C = \underset{\substack{\downarrow \\ N(E_F)}}{\gamma} T + \underset{\substack{\uparrow \\ T_D}}{\beta} T^3$$



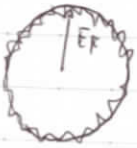
	T_D(K)
Li	400
Na	150
Be	1000
C(diamond)	1860
Si	625





$$E \propto (k_BT)^4 \qquad C \propto T^3$$

$$= k_BT \cdot \underset{\uparrow\ \text{体積}}{(k_BT)^3}$$



$$E \sim (k_BT)^2 \qquad C \propto T$$

1.7 非調和性と熱膨張

$$V = V_0 + \underbrace{\sum_{i,j} \frac{\partial^2 V}{\partial u_i \partial u_j} u_i u_j}_{\text{二次}} + \underbrace{\sum_{i,j} \frac{\partial^3 V}{\partial u_i \partial u_j \partial u_k} u_i u_j u_k}_{\text{三次}} + \cdots$$

$$V = V_0 + \frac{1}{2} \sum_i \alpha (u_{i+1} - u_i)^2$$

三次の項が入ると

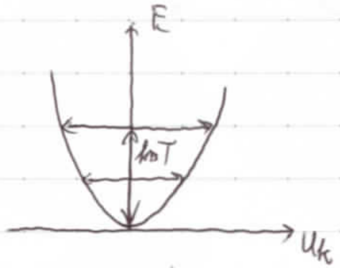
3フォノンプロセス

$$\omega_1 + \omega_2 = \omega_3$$

$$q_1 + q_2 = q_3 + \underset{\sim}{G}$$

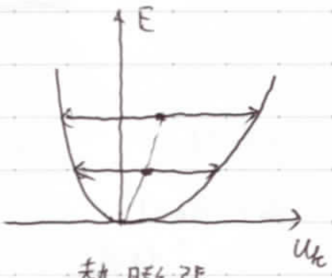
モーメント保存　↑ ウムクラップ

有限の平均自由行程



平均位置変わらない

調和項のみ



熱膨張

非調和項

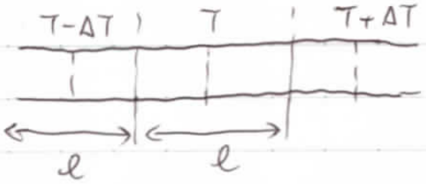
1.8 熱伝導

$$\underset{\text{J/m}^2\cdot\text{s}}{j} = \underset{\downarrow}{K}\ \underset{\text{K/m}}{\frac{dT}{dx}}$$

$$\text{J m}^{-1}\text{s}^{-1}\text{K}^{-1} = \text{W m}^{-1}\text{K}^{-1}$$

$$K \sim \underset{\substack{\uparrow \\ \text{ガラス}}}{\overset{\text{室温}}{1}} - 100$$

一次元



平均自由行程 $l = v\tau$

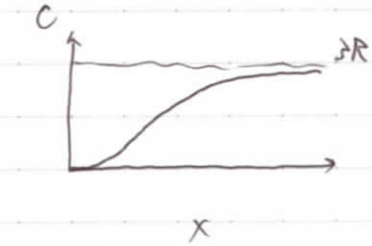
$$j = c\frac{dT}{dx}\,l\cdot v\cdot 1^2$$

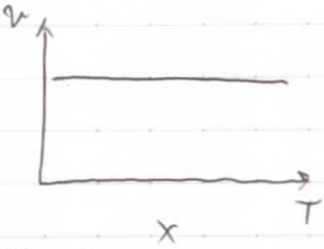
$$= \underset{\substack{\shortparallel \\ K}}{cvl}\,\frac{dT}{dx}$$

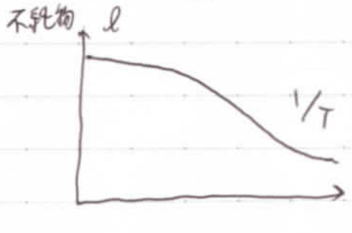
3次元 $c\ v_x\ v_x\tau\ \frac{dT}{dx}$

$$\langle v_x^2\rangle = \frac{1}{3}\langle v^2\rangle$$

$$K = \frac{1}{3}cv^2\tau = \frac{1}{3}cvl$$







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